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Improved Lee, Oehme and Yang approximation.

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Abstract

The Lee, Oehme and Yang (LOY) theory of time evolution in two state subspace of states of the complete system is discussed. Some inconsistencies in assumptions and approximations used in the standard derivation of the LOY effective Hamiltonian, H_{LOY}, governig this time evolution are found. Eliminating these inconsistecies and using the LOY method, approximate formulae for the effective Hamiltonian, $H_{\parallel}$, governing the time evolution in this subspace (improving those obtained by LOY) are derived. It is found, in contradistinction to the standard LOY result, that in the case of neutral kaons ($< K_0|H_{\parallel}|K_0 > - < \overline{K}_0|H_{\parallel}|\overline{K}_0 >$), cannot take the zero value if the total system the preserves CPT–symmetry. Within the use of the method mentioned above formulae for $H_{\parallel}$ acting in the three state (three dimensional) subspace of states are also found.

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1 Introduction.

In the quantum decay theory of multiparticle complexes, properties of the transition amplitudes

$$A_{u_j;\psi}(t) =< u_j|\psi;t > \tag{1}$$

are usually analysed. Here vectors $\{|u_j>\}_{j\in U}$ represent the unstable states of the system considered, $< u_j|u_k >= \delta_{jk}$, and $|\psi;t>$ is the solution of the Schrödinger equation (we use $\hbar = c = 1$ units)

$$i\frac{\partial}{\partial t}|\psi;t >= H|\psi;t >, \tag{2}$$

having the following form

$$|\psi;t >= \sum_{j\in U} a_j(t)|u_j > + \sum_J f_J(t)|\phi_J >, \tag{3}$$

where vectors $|\phi_J>$ describe the states of decay products, $< u_j|\phi_J >= 0$ for every $j \in U$. The initial condition for Eq (2) in the case considered is usually assumed to be

$$\begin{aligned} |\psi;t = t_0 \equiv 0 > &\stackrel{\text{def}}{=} |\psi >\equiv \sum_{j\in U} a_j|u_j >, \\ f_J(t=0) &= 0. \end{aligned} \tag{4}$$

In Eq (2) H denotes the complete (full), selfadjoint Hamiltonian of the system. We have $|\psi;t >= e^{-itH}|\psi >$. It is not difficult to see that this property and hermiticity of H imply that

$$A_{u_j,u_j}(t)^* = A_{u_j;u_j}(-t). \tag{5}$$

Therefore, the decay probability of an unstable state (usually called the decay law), i.e., the probability for a quantum system to remain in its initial state $|\psi >\equiv |u_j>$

$$p_{u_j}(t) \stackrel{\text{def}}{=} |A_{u_j;u_j}(t)|^2 \equiv |a_j(t)|^2, \tag{6}$$

must be an even function of time:

$$p_{u_j}(t) = p_{u_j}(-t). \tag{7}$$

This last property suggests that in the case of the unstable states prepared at some instant t_0, say $t_0 = 0$, the initial condtion (4) for the evolution equation (2) should be formulated more precisely. Namely, from (7) it follows that the probabilities of finding the system in the decaying state $|u_j>$ at the instant, say $t = T \gg t_0 \equiv 0$, and at the instant $t = -T$ are the same. Of course, this can never occur. In almost all experiments in which the decay law of a given unstable particle is investigated this particle is created at some instant of time, say t_0, and this instant of time is usually considered as the initial instant for the problem. From the property (7) it follows that the instantaneous creation of the unstable particle is impossible. For the observer, the creation of this particle (i.e., the preparation of the state, $|u_j>$, representing the decaying particle) is practically instantaneous. What is more, using suitable detectors he is usually able to prove that it did not exist at times $t < t_0$. Therefore, if one looks for the solutions of the Schrödinger equation (2) describing properties of the unstable states prepared at some initial instant t_0 in the system, and if one requires these solutions to reflect situations described above, one should completement initial conditions (4) for Eq (2) by assuming additionally that

$$a_j(t < t_0) = 0, \quad (j \in U), \tag{8}$$

and that, for the problem, time t varies from $t = t_0 > -\infty$ to $t = +\infty$ only.

Amplitudes of type $a_j(t)$ can be calculated directly by solving the evolution equation (2), or by using the Schrödinger–like evolution equation governing the time evolution in a subspace spanned by the set of vectors $\{|u_j>\}_{j\in U}$. Searching for the properties of two particle subsystems one usually uses the following equation of the type mentioned [1] — [19] instead of Eq (2),

$$i\frac{\partial}{\partial t}|\psi; t>_{\|} = H_{\|}|\psi; t>_{\|}, \tag{9}$$

where by $H_{\|}$ we denote the effective nonhermitian Hamiltonian,

$$H_{\|} \equiv M - \frac{i}{2}\Gamma, \tag{10}$$

and

$$M = M^{+}, \quad \Gamma = \Gamma^{+}, \tag{11}$$

are (2×2) matrices, acting in a two–dimensional subspace $\mathcal{H}_{\|}$ of the total state space $\mathcal{H}$. M is called the mass matrix, Γ is the decay matrix [1] — [7]. The standard method of derivation of such a $H_{\|}$ is based on a modification of Weisskopf–Wigner (WW) approximation [20]. Lee, Oehme and Yang (LOY) adapted the WW aproach to the case of a two particle subsystem [1] — [6] to obtain their effective Hamiltonian $H_{\|} \equiv H_{LOY}$. Almost all properties of the neutral kaon complex, or another two state subsystem, can be described by solving Eq (9) [1] — [19], with the initial condition corresponding to (4) and (8)

$$\begin{aligned} |\psi; t = t_0 >_{\|} &\equiv |\psi >_{\|}, \\ \| \ |\psi; t = t_0 >_{\|} \ \| &= 1, \ \ |\psi; t < t_0 >_{\|} = 0, \end{aligned} \tag{12}$$

for $|\psi; t >_{\|}$ belonging to the subspace $\mathcal{H}_{\|} \subset \mathcal{H}$ spanned, e.g., by orthonormal neutral kaons states $|K_0 >$, $|\overline{K}_0 >$, and so on, (then states corresponding to the decay products belong to $\mathcal{H} \ominus \mathcal{H}_{\|} \stackrel{\text{def}}{=} \mathcal{H}_{\perp}$),

$$|\psi >_{\|} \equiv a_1 |\mathbf{1} > + a_2 |\mathbf{2} >, \tag{13}$$

and $|\mathbf{1} >$ stands for the vectors of the $|K_0 >$, $|B_0 >$, etc., type and $|\mathbf{2} >$ denotes states of $|\overline{K}_0 >$, $\overline{B}_0 >$ type, $< \mathbf{j} | \mathbf{k} > = \delta_{jk}$, $j, k = 1, 2$.

The old, as well as the more recent [5] — [7] experimental tests of the CP–noninvariance and of the CPT–invariance in the neutral kaon system need a correct interpretation of the measured CP– nad CPT–violation parameters. In the large literature, all CP– and CPT–violation parameters in the neutral kaon and similar complexes are expressed in terms of matrix elements of $H_{\|} \equiv H_{LOY}$. On the other hand, in some papers the correctness and selfconsistency of the LOY approximation is questioned [10] — [19], [21]. Therefore it seems to be important to examine in detail the derivation of the formulae for H_{LOY}.

The paper is organized as follows. We begin with the discussion of the Lee, Oehme and Yang theory: Deriving formulae for matrix elements of H_{LOY} in Sec. 2 we will apply the method used in [3] with insignificant modifications. In Sec. 3 within the use of the same "recipe" as in Sec. 2, instead of the formulae for matrix elements, the formula for the complete operator H_{LOY} is derived and the questionable points of the LOY approach are found. Namely in [1, 2] terms of type $< \mathbf{j} | H^{(1)} | \mathbf{k} >$, where $H^{(1)}$ denotes a small perturbation and $j, k = 1, 2$, are neglected in the initial equations for amplitudes $a_j(t)$.

The aim of this paper is to show that taking into account such terms with the use of all remaining LOY assumptions lead to the effective Hamiltonian $H_{\|}$, which differs from H_{LOY}. This $H_{\|} = H_{LOY}^{Imp}$ improving H_{LOY} is also found in this Section. The improved LOY method is used in Sec. 4 to derive the effective Hamiltonian $H_{\|}$ governing the time evolution in the three dimesional (three state) subspace of states. Sec. 5 contains a summary and conclusions.

2 Analysis of steps leading to the standard formulae for H_{LOY}.

2.1 Detailed derivation of H_{LOY}.

Let us now consider all the steps leading to the formulae for the matrix elements of H_{LOY} in detail. As it has already been mentioned, the source of the LOY model for the decay of neutral kaons is the well known Weisskopf–Wigner approach to the description of unstable states [20]. Within this approach, the Hamiltonian H for the problem is divided into two parts $H^{(0)}$ and $H^{(1)}$:

$$H \;=\; H^{(0)} + H^{(1)}, \tag{14}$$

such that $|K_0> \equiv |\mathbf{1}>$ and $|\overline{K}_0> \equiv |\mathbf{2}>$ are discrete eigenstates of $H^{(0)}$ for the 2–fold degenerate eigenvalue m_0,

$$H^{(0)}|\mathbf{j}> = m_0|\mathbf{j}>, \;\; j = 1, 2; \tag{15}$$

and $H^{(1)}$ induces the transitions from these states to other (unbound) eigenstates $|\varepsilon, J>$ of $H^{(0)}$ (here J denotes such quantum numbers as charge, spin, etc.), and, consequently, also between $|K_0>$ and $|\overline{K}_0>$. So, the problem which one usually considers is the time evolution of an initial state, which is a superposition of $|\mathbf{1}>$ and $|\mathbf{2}>$ states.

In the kaon rest–frame, this time evolution for $t \geq t_0 \equiv 0$ is governed by the Schrödinger equation (2), whose solutions $|\psi; t>$ have the following form

$$|\psi; t> = a_1(t)|\mathbf{1}> + a_2(t)|\mathbf{2}> + \sum_{J,\varepsilon} F_J(\varepsilon; t)|\varepsilon, J>, \tag{16}$$

and

$$|a_1(t)|^2 + |a_2(t)|^2 + \sum_{J,\varepsilon} |F_J(\varepsilon, t)|^2 = 1. \tag{17}$$

Here $|F_J;t>\equiv\sum_\varepsilon F_J(\varepsilon;t)|\varepsilon,J>$ represents the decay products in the channel J; $<\varepsilon,J|\mathbf{k}>=0$, $k=1,2$; $<\varepsilon',L|\varepsilon,N>=\delta_{LN}\delta(\varepsilon-\varepsilon')$.

From the Schrödinger equation (2) the following equations for amplitudes $a_1(t)$, $a_2(t)$ and $F_J(\varepsilon;t)$ can be obtained

$$\begin{aligned} i\frac{\partial}{\partial t}a_k(t) &= m_0 a_k(t)+\sum_{l=1}^{2}H^{(1)}_{kl}a_l(t) \\ &+ \sum_{J,\varepsilon}H^{(1)}_{kJ}(\varepsilon)F_J(\varepsilon;t), \quad (_{k=1,2};\ \ t\geq 0) \qquad (18) \\ i\frac{\partial}{\partial t}F_J(\varepsilon;t) &= \varepsilon F_J(\varepsilon;t)+\sum_{k=1,2}H^{(1)}_{Jk}(\varepsilon)a_k(t) \\ &+ \sum_{L,\varepsilon'}F_L(\varepsilon';t)H^{(1)}_{J,L}(\varepsilon,\varepsilon'), \quad (t\geq 0), \qquad (19) \end{aligned}$$

where $H^{(1)}_{kJ}(\varepsilon)=(H^{(1)}_{Jk}(\varepsilon))^*=<\mathbf{k}|H^{(1)}|\varepsilon,J>$, $(k=1,2)$, are the matrix elements responsible for the decay, $H^{(1)}_{J,L}(\varepsilon,\varepsilon')=<J,\varepsilon|H^{(1)}|\varepsilon',L>$, and $H^{(1)}_{kl}$ $=<\mathbf{k}|H^{(1)}|\mathbf{l}>$; $k,l=1,2$. These equations are exact. In agreement with (12), the boundary conditions for Eqs (18), (19) are following:

$$a_k(0)=a_k,\ \ a_k(t<0)=0,\ \ (_{k=1,2}), \qquad (20)$$

and

$$F_J(\varepsilon;t=0)=0, \qquad (21)$$

so

$$|a_1|^2+|a_2|^2=1. \qquad (22)$$

In the WW approach to solving the Schrödinger equation (2) it is required that the martix elements of type $H^{(1)}_{jk}$, $H^{(1)}_{kJ}(\varepsilon)$, etc., should be suitably small [20]. From [1, 2] and [3] one can conclude that the LOY modification of the WW method consists of assuming that, among others,

$$\begin{aligned} \sum_{k=1,2}|H^{(1)}_{jk}| &\ll m_0, \quad (_{j=1,2}), \qquad (23) \\ \sum_{J,\varepsilon}|H^{(1)}_{kJ}(\varepsilon)| &\ll m_0, \quad (_{k=1,2}), \qquad (24) \\ \sum_{l=1,2}|H^{(1)}_{kl}| &\ll \sum_{J,\varepsilon}|H^{(1)}_{kJ}(\varepsilon)|, \quad (_{k=1,2}), \qquad (25) \end{aligned}$$

and

$$\sum_{L,\varepsilon'} |H^{(1)}_{J,L}(\varepsilon,\varepsilon')| \ll \sum_{k=1,2} |H^{(1)}_{Jk}(\varepsilon)|, \tag{26}$$

for every J.

Assumptions of type (23) — (26) were used by LOY in order to replace the exact equations of type (18), (19) by approximate equations (18) — (20) considered in [1] (see also [3], Chap. 5, Appendix 1, Equations (A1.4) — (A1.6)). The mentioned LOY equations are equivalent to the following approximate ones, which are valid if the requirements (23) — (26) hold

$$i\frac{\partial}{\partial t}a_k(t) = m_0 a_k(t) + \sum_{J,\varepsilon} H^{(1)}_{kJ}(\varepsilon) F_J(\varepsilon;t), \quad (_{k=1,2}), \tag{27}$$

$$i\frac{\partial}{\partial t}F_J(\varepsilon;t) = \varepsilon F_J(\varepsilon;t) + \sum_{k=1,2} H^{(1)}_{Jk}(\varepsilon) a_k(t). \tag{28}$$

Eqs (27), (28) differ from LOY Eqs (18) — (20) of [1], among others, in the first componets of their right sides . Such components are absent in the LOY equations. This difference is caused by using the interaction representation in [1] and rescaling the energy, ε: $\varepsilon \to \omega = \varepsilon - m_0$, which means that the zero of energy is taken to be the rest energy of K. Another difference is the following: In the right sides of the LOY equations factors of type $e^{\pm i\omega t}$ are present. They are absent in Eqs (27), (28). The presence of these factors in LOY equations is due to the use of the interaction representation. Nevertheless, the mathematical equivalence of Eqs (27), (28) and Eqs (18) — (20) of [1] is rigorous.

The WW theory states that under the assumptions (23) — (26), the actual contribution of the second component on the rigth side of Eq (27) into the amplitude $a_k(t)$ is very small. From [1, 2, 3] one can conclude that in the LOY treatment of the problem this contribution resolves itself into adding some small complex number, say Λ, to the parameter m_0, such that $|\Lambda| \ll m_0$, and $\mathrm{Im.}\,\Lambda = -\frac{\gamma}{2} < 0$. Simply, the interactions which are responsible for the presence of this second component in the considered equation slightly shift the level m_0: $m_0 \to m_0 + \Lambda$. So, the replacement of Eq (27) by the following approximate one seems to be justifiable

$$i\frac{\partial}{\partial t}a_k(t) \simeq (m_0 + \Lambda) a_k(t), \quad (_{k=1,2;\ t>0}). \tag{29}$$

which means that under the conditions (23) — (26), the amplitudes $a_k(t)$ should take the following form

$$a_k(t) \simeq e^{-i(m_0+\Lambda)t} a_k, \quad (k=1,2;\ t>0), \tag{30}$$

Therefore, when one looks for the solutions of Eq (27), the use of the assumption

$$\frac{a_1(t)}{a_1} = \frac{a_2(t)}{a_2} = e^{-i(m_0+\Lambda)t}, \quad (t>0), \tag{31}$$

is considerd to be obvious. This assumption is equivalent to the LOY assumption (21) of [1] (or, (A1.1) in [3], Appendix of Chap. 5), which is easily seen if (31) is rewritten in the LOY manner:

$$|\psi; t>_{\|} = e^{-i(m_0+\Lambda)t} |\psi>_{\|}, \quad (t>0), \tag{32}$$

where

$$|\psi; t>_{\|} = a_1(t)|\mathbf{1}> + a_2(t)|\mathbf{2}> . \tag{33}$$

The assumption (31) (or (32)) is crucial to the LOY method and it is the essence of the approximation which was made in [1, 3]. It determines all the properties of the effective Hamiltonian H_{LOY} governing the time evolution in a two state subspace.

Defining

$$F_J(\varepsilon; t) \stackrel{\text{def}}{=} e^{-i\varepsilon t} \tilde{F}_J(\varepsilon; t), \tag{34}$$

Eq (28) can be transformed into

$$\begin{aligned} i\frac{\partial}{\partial t}\tilde{F}_J(\varepsilon; t) &= \sum_{k=1,2} e^{i\varepsilon t} H^{(1)}_{Jk}(\varepsilon) a_k(t), \\ \tilde{F}_J(\varepsilon; t=0) &= 0, \end{aligned} \tag{35}$$

which can easily be solved and leads to the following solution for $F_J(\varepsilon; t)$ with $t \geq 0$:

$$F_J(\varepsilon; t) = (-i) \sum_{k=1,2} \int_0^t e^{-i\varepsilon(t-\tau)} H^{(1)}_{Jk}(\varepsilon) a_k(\tau) d\tau. \tag{36}$$

Now, one can eliminate $F_J(\varepsilon; t)$ from Eq (27) by substituting (36) back into Eq (27). This leads to the following equation, eg., for $a_1(t)$ with $t \geq 0$,

$$i\frac{\partial}{\partial t} a_1(t) = m_0 a_1(t) - i \sum_{k=1,2} \sum_{J,\varepsilon} \int_0^t e^{-i\varepsilon(t-\tau)} H^{(1)}_{1J}(\varepsilon) H^{(1)}_{Jk}(\varepsilon) a_k(\tau)\, d\tau. \tag{37}$$

Next, inserting (31) into (37) one finds the following equation for $a_1(t)|_{t>0}$

$$\Big\{i\frac{\partial}{\partial t}-m_0\Big\}a_1(t) \;=\; (-i)\sum_{k=1,2}\Big\{\sum_{J,\varepsilon}\int_0^t e^{-i(\varepsilon-m_0-\Lambda)(t-\tau)}H^{(1)}_{1J}(\varepsilon)\times \\ \times H^{(1)}_{Jk}(\varepsilon)d\tau\Big\}a_k(t). \qquad (38)$$

The main properties of the quasistationary, or bound states manifest themselves at times $t \gg t_0 = 0$, where t_0 is the moment of their preparation. Therefore, it is reasonable to replace the upper limit $t < \infty$ of the integrals in Eq (38) by $t \to \infty$. Also, as it was mentioned, Λ is a very small number. So, the formulae for the lowest notrivial order of the matrix elements h^{LOY}_{jk} of H_{LOY}, are obtained by putting $\Lambda = 0$ under the integrals in Eq (38) and then evaluating these integrals and passing to the limit $t \to \infty$. (In this case these matrix elements will be denoted by $h^{LOY(0)}_{jk}$, and the the effective Hamiltonian by $H^{(0)}_{LOY}$). Such a treatment of Eq (38) gives (compare [3])

$$\Big\{i\frac{\partial}{\partial t}-m_0\Big\}a_1(t) \;\simeq\; -\sum_{k=1,2}\Big\{\lim_{t\to\infty}\sum_{J,\varepsilon}\frac{1-e^{-i(\varepsilon-m_0)t}}{\varepsilon-m_0}\times \\ \times H^{(1)}_{1J}(\varepsilon)H^{(1)}_{Jk}(\varepsilon)\Big\}\,a_k(t). \qquad (39)$$

where $t \gg t_0 = 0$. This last equation can be rewritten as follows

$$\Big\{i\frac{\partial}{\partial t}-m_0\Big\}a_1(t) = -\Sigma^{(0)}_{11}(m_0)a_1(t) - \Sigma^{(0)}_{12}(m_0)a_2(t), \qquad (40)$$

where $t \gg t_0 = 0$, and

$$\Sigma^{(0)}_{jk}(x) = \sum_{J,\varepsilon} H^{(1)}_{jJ}(\varepsilon)\frac{1}{\varepsilon - x - i0}H^{(1)}_{Jk}(\varepsilon) =< \mathbf{j}|\Sigma^{(0)}(x)|\mathbf{k} > .\ \ {}_{(j,k=1,2)}. \qquad (41)$$

A similar equation can be obtained for the amplitude $a_2(t)$. This means that the matrix elements $h^{LOY(0)}_{jk} =< \mathbf{j}|H^{(0)}_{LOY}|\mathbf{k} >$ equal

$$h^{LOY(0)}_{jk} = m_0\delta_{jk} - \Sigma^{(0)}_{jk}(m_0) \equiv M_{jk} - \frac{i}{2}\Gamma_{jk},\ \ {}_{(j,k=1,2)}, \qquad (42)$$

i.e., exactly as in [1] — [8].

These formulae are the frame for almost all calculations of the parameters characterizing the properties of the neutral kaons complex and other two level subsystems [8, 9].

2.2 Operator form of H_{LOY}.

Defining projectors

$$P = |\mathbf{1}><\mathbf{1}| + |\mathbf{2}><\mathbf{2}|, \tag{43}$$

$$Q = I - P, \tag{44}$$

$$[P, H^{(0)}] = 0, \quad [P, H^{(1)}] \neq 0, \tag{45}$$

(where I is the unit operator in $\mathcal{H}$), allows us to rewrite the $H^{(0)}_{LOY}$ in a compact form which is sometime more convenient than the standard one (42):

$$H^{(0)}_{LOY} = m_0 P - \Sigma^{(0)}(m_0) \equiv M^{LOY} - \frac{i}{2}\Gamma^{LOY}, \tag{46}$$

where

$$\Sigma^{(0)}(x) = PHQ\frac{1}{H^{(0)} - x - i0}QHP. \tag{47}$$

The $H^{(0)}_{LOY}$ acts in a two dimensional subspace $\mathcal{H}_{\|}$ of $\mathcal{H}$. This $\mathcal{H}_{\|}$ can be defined by means of the projector P in the following way

$$\mathcal{H}_{\|} \stackrel{\text{def}}{=} P\mathcal{H} \ni |\psi; t>_{\|}, \tag{48}$$

where

$$|\psi; t>_{\|} \equiv P|\psi; t> . \tag{49}$$

The projector Q defines the subspace of decay products $\mathcal{H}_{\perp}$:

$$\mathcal{H}_{\perp} \stackrel{\text{def}}{=} Q\mathcal{H} \equiv \mathcal{H} \ominus \mathcal{H}_{\|} \ni |\psi; t>_{\perp}, |F_J; t>, \tag{50}$$

where

$$|\psi; t>_{\perp} \stackrel{\text{def}}{=} Q|\psi; t> . \tag{51}$$

Note that assumptions used in [1, 3] lead to following the property

$$\sum_{j=1,2} |\mathbf{j}><\mathbf{j}| + \sum_{J,\varepsilon} |\varepsilon, J><J, \varepsilon| = I.$$

This means that the standard LOY approach enable us to conclude that

$$I - P \equiv Q \equiv \sum_{J,\varepsilon} |\varepsilon, J><J, \varepsilon|.$$

One should stress it that in a general case this last relation need not be valid and it will not be used in subsequent Sections of this paper.

2.3 CPT transformation properties of H_{LOY}.

Usually, in the LOY and related approaches, it is assumed that the free Hamiltonian $H^{(0)}$ is CPT–invariant [1] — [7]:

$$[\Theta, H^{(0)}] = 0, \tag{52}$$

where Θ is the antiunitary operator:

$$\Theta \stackrel{\rm def}{=} \mathcal{CPT}, \tag{53}$$

and $\mathcal{C}$ is the charge conjugation operator, $\mathcal{P}$ — space inversion, and the antiunitary operator $\mathcal{T}$ represents the time reversal operation. (Basic properties of anti–linear and linear operators, their products and commutators are described, eg., in [22, 23, 24]).

Using, e.g., the following phase convention [2] — [6]

$$\Theta|\mathbf{1}> \stackrel{\rm def}{=} -|\mathbf{2}>, \;\; \Theta|\mathbf{2}> \stackrel{\rm def}{=} -|\mathbf{1}>, \tag{54}$$

which means that the subspace of neutral kaons $\mathcal{H}_{\|}$ is assumed to be invariant under Θ:

$$[\Theta, P] = 0, \tag{55}$$

one easily finds from (42) that in the case of the CPT–invariant interactions

$$[\Theta, H^{(1)}] = 0, \tag{56}$$

i.e., in the CPT–invariant system

$$[\Theta, H] = 0, \tag{57}$$

the diagonal matrix elements of $H^{(0)}_{LOY}$ must be equal:

$$h^{LOY(0)}_{11} = h^{LOY(0)}_{22}. \tag{58}$$

This is the standard result of the LOY approach and this is the picture which one meets in the literature [1] — [13].

3 Improved LOY approximation.

3.1 General considerations.

In the previous Section the coupled system equations (18), (19) for number functions (amplitudes) $a_k(t)$, $F_J(\varepsilon;t)$ have been analysed. While considering each of the equations separately there is a danger of overlooking some common, global properties of a such system and thus similar properties of the physical system under consideration. It seems that a complex look at the equations governig the time evolution in the subsystem considered should either confirm all the conclusions and formulae derived above or show that they are incorrect. It should also indicate all the questionable steps in the standard derivation of H_{LOY}. So, let us consider the evolution equations for the components $|\psi;t>_{\|}$ (13), (49) and for $|\psi;t>_{\perp}$ (51) of the state vector $|\psi;t>$ (16) instead of the system equations for number functions $a_k(t)$, $F_J(\varepsilon;t)$. Using projection operators P and Q, (43), (44), one can obtain from the Schrödinger equation (2) for the state vector $|\psi;t>$ two equations for its orthogonal components $|\psi;t>_{\|}$ (13), (49) and $|\psi;t>_{\perp}$ (51) valid for $t \geq t_0 = 0$:

$$
\begin{aligned}
i\frac{\partial}{\partial t}|\psi;t>_{\|} &= PHP|\psi;t>_{\|} + PHQ|\psi;t>_{\perp}, && (59)\\
&\equiv \left\{m_0 P + PH^{(1)}P\right\}|\psi;t>_{\|} + PH^{(1)}Q|\psi;t>_{\perp}, && (60)\\
i\frac{\partial}{\partial t}|\psi;t>_{\perp} &= QHQ|\psi;t>_{\perp} + QHP|\psi;t>_{\|}, && (61)\\
&\equiv QHQ|\psi;t>_{\perp} + QH^{(1)}P|\psi;t>_{\|}, && (62)
\end{aligned}
$$

with the initial conditions (12), (13) and (21), which are equivalent to the following one

$$|\psi;t=0>_{\perp} = 0. \qquad (63)$$

Let us consider a general case of Eqs (59) and (61). According to the LOY method, as in the usual single line width problem of atomic transitions [20], the contribution arising from decay products $|\psi;t>_{\perp} \in \mathcal{H}_{\perp}$ into the time derivative $i\frac{\partial}{\partial t}|\psi;t>_{\|}$ in Eq (59) should be eliminated. Within this method, assuming that such a contribution is suitably small, one requires $i\frac{\partial}{\partial t}|\psi;t>_{\|}$ to be expressed in terms of $|\psi;t>_{\|}$ only. From the superposition principle Lee and Wu conclude in [2] that such an expression should be time independent

and linear. Using this observation we find that to fulfill this requirement, if the transitions from the subspace of decay products $\mathcal{H}_\perp \ni |\psi; t >_\perp$ are sufficiently weak (see ([25])), i.e., if for every finite $t \geq 0$,

$$\| PHQ|\psi; t >_\perp \| \ll \| PHP|\psi; t >_\| \|, \tag{64}$$

the following substitution into Eq (59) should be made

$$PHQ|\psi; t >_\perp = PH^{(1)}Q|\psi; t >_\perp \equiv V_\| |\psi; t >_\|, \tag{65}$$

where $V_\|$ is in general an linear and nonhermitian operator (a nonhermitian matrix) acting in the subspace $\mathcal{H}_\| \ni |\psi; t >_\|$. It is additionaly assumed in [2] that an operator of this type should be time independent. Then, to a very good approximation, Eqs (59), (60) take the required form

$$\begin{aligned} i\frac{\partial}{\partial t}|\psi; t >_\| &= \Big\{PHP + V_\|\Big\}|\psi; t >_\| & (66) \\ &\equiv \Big\{m_0 P + PH^{(1)}P + V_\|\Big\}|\psi; t >_\| \, . & (67) \end{aligned}$$

This (within the use of the LOY assumption of time independence $V_\|$) means that one should expect the solutions of (59), (60) to have the exponential, similar to (31) and (32), form:

$$\begin{aligned} |\psi; t >_\| &= e^{-it(PHP + V_\|)}|\psi; t = 0 >_\| & (68) \\ &\equiv e^{-it(m_0 P + PH^{(1)}P + V_\|)}|\psi; t = 0 >_\|, & (69) \end{aligned}$$

and, as it has been done in the LOY theory, such a form of $|\psi; t >_\|$ can be used for the calculation of the effective Hamiltonian $H_\|$,

$$\begin{aligned} H_\| &\stackrel{\text{def}}{=} PHP + V_\| & (70) \\ &\equiv m_0 P + PH^{(1)}P + V_\|, & (71) \end{aligned}$$

governing the time evolution in the subspace considered.

Solving Eq (61) one can eliminate $|\psi; t >_\perp$ from Eq (59) by substituting the solution of Eq (61) back into Eq (59). Looking for this solution we will follow the method used to solve Eq (19) in Sec. 2. Namely, by means of the substitution

$$|\widetilde{\psi; t} >_\perp \stackrel{\text{def}}{=} e^{+itQHQ}|\psi; t >_\perp, \;\; (t \geq 0), \tag{72}$$

Eq (61) can be replaced by the following one

$$\begin{aligned} i\frac{\partial}{\partial t}|\widetilde{\psi;t}>_{\perp} &= e^{+itQHQ}{}_{QHP}|\psi;t>_{\parallel}, \ \ (t\geq 0), \\ |\widetilde{\psi;t=0}>_{\perp} &= 0. \end{aligned} \tag{73}$$

It is easy to solve this equation. Using its solution one finds

$$|\psi;t>_{\perp} = -i\int_0^t e^{-i(t-\tau)QHQ}{}_{QHP}|\psi;\tau>_{\parallel}\ d\tau, \ \ (t\geq 0), \tag{74}$$

which is in perfect agreement with the result (36) in Sec. 2.

Substituting (74) back into Eq (59) gives for $t \geq 0$:

$$i\frac{\partial}{\partial t}|\psi;t>_{\parallel} = PHP|\psi;t>_{\parallel} -i\int_0^t PHQe^{-i(t-\tau)QHQ}{}_{QHP}|\psi;\tau>_{\parallel}\ d\tau, \tag{75}$$

which is an analogon of Eq (37) in Sec. 2. Notice that in contradistinction to Eq (37) mentioned, Eq (75) is exact. (In the literature, equations of type Eq (37) are called "master equation" [26], or Krolikowski–Rzewuski equation for the distinguished component of a state vector [27] — [31]).

Now inserting the expected exponential form of $|\psi;t>_{\parallel}$ (68) into Eq (75) and, taking into account (as in Sec. 2) the fact that all characteristic properies of bound, or quasistationary states manifest themselves at times $t \gg t_0$, practically for $t \to \infty$, (here t_0 is the moment of the preparation of the subsystem considered), one obtains, to a very good approximation

$$\begin{aligned} i\frac{\partial}{\partial t}|\psi;t>_{\parallel} \ \cong \ PHP|\psi;t>_{\parallel} -i\Big\{\lim_{t\to\infty}\int_0^t \Big[PHQe^{-i(t-\tau)QHQ}{}_{QHP}\times \\ \times e^{i(t-\tau)(PHP+V_{\parallel})}\Big]\,d\tau\Big\}\,|\psi;t>_{\parallel}, \end{aligned} \tag{76}$$

(where $t \gg t_0 = 0$), which is analogous to Eqs (38), Eq (39).

On the other hand, if the solution (74) of Eq (61) is directly substituted into Eq (65), then one immediatelly finds that

$$V_{\parallel}|\psi;t> \stackrel{\text{def}}{=} -i\int_0^t PHQe^{-i(t-\tau)QHQ}{}_{QHP}|\psi;\tau>_{\parallel}\ d\tau. \tag{77}$$

Now keeping in mind the motivation in relation to time t presented before Eq (76) and inserting the form (68) of $|\psi;t>_{\parallel}$ predicted by the LOY approach

into (77) one obtains the following relation which is valid to a very good approximation for $t \gg t_0 = 0$,

$$V_{\|}|\psi; t>_{\|} \;\cong\; -i\Big\{ \lim_{t\to\infty} \int_0^t \Big[PHQe^{-i(t-\tau)QHQ}{}_{QHP} \times \\ \times e^{i(t-\tau)(PHP+V_{\|})}\, d\tau\Big\}\, |\psi; t>_{\|}\,. \qquad (78)$$

From this equation, or from Eq (76) one can infer that the operator (the matrix) $V_{\|}$ can be obtained by solving the nonlinear equation

$$V_{\|} = -i \lim_{t\to\infty} \int_0^t PHQe^{-i(t-\tau)QHQ}{}_{QHP} e^{i(t-\tau)(PHP+V_{\|})}\, d\tau. \qquad (79)$$

So, the consistently applied LOY method leads to the nonlinear equation for the effective Hamiltonian $H_{\|}$, (70), governing the time evolution in the subspace $\mathcal{H}_{\|}$. Similar equations one can meet in theories of equations of the "master equation" type, [26]— [31].

Solutions of Eq (79) can be found, e.g., by means of the iteration method. Putting in (79) (see [27])

$$V_{\|}^{(n+1)} = -i \lim_{t\to\infty} \int_0^t PHQe^{-i(t-\tau)QHQ}{}_{QHP} e^{i(t-\tau)(PHP+V_{\|}^{(n)})}\, d\tau, \qquad (80)$$

one can express $V_{\|}$ as follows

$$V_{\|} = \lim_{n\to\infty} V_{\|}^{(n)}. \qquad (81)$$

Taking into account the fact that the contribution of the component $|\psi; t>_{\perp}$ into Eq (59) for $|\psi; t>_{\|}$ is (by the assumption (64)) very small, and therefore that the matrix elements of the operator $V_{\|}$, (65), expressing this contribution should be very small also, it seems reasonable to assume that

$$V_{\|}^{(0)} = 0. \qquad (82)$$

Such an assumption corresponds with the similar one exploited in the LOY approach, i.e., which is made in Sec. 2 for the parameter Λ appearing in formulae (29) — (32), where the final formulae for the matrix elements of H_{LOY} were obtained by assuming that $\Lambda = 0$ (see Eq (39)). Therefore the identification of the approximate solutions $V_{\|}^{(1)}$ of Eq (80),

$$V_{\|}^{(1)} = -i \lim_{t\to\infty} \int_0^t PHQe^{-i(t-\tau)QHQ}{}_{QHP} e^{-i(t-\tau)PHP}\, d\tau, \qquad (83)$$

with the LOY effective Hamiltonian $H_{\|} = H_{LOY}$, (or with the improved LOY effective Hamiltonian $H_{\|} = H_{LOY}^{Imp}$), by the relation (70),

$$H_{LOY}\ (H_{LOY}^{Imp})\ =\ H_{\|}^{(1)} \equiv PHP + V_{\|}^{(1)}, \tag{84}$$

seems to be well–grounded.

Now let us analyse more carefuly relations (65) and (77). From these relations it follows that in fact the supposition made in [2] that $i\frac{\partial}{\partial t}|\psi;t>_{\|}$ can be expressed in terms of $|\psi;t>_{\|}$ only by means of time independent coefficients, that is that $V_{\|}$ should be time independent, in general is not true. Analysing the mentioned formulae and the initial condition (63) one finds that

$$V_{\|} \equiv V_{\|}(t), \quad \text{and} \quad V_{\|}(t=0) = 0, l;\ V_{\|}(t>0) \neq 0. \tag{85}$$

This means that the expected in the LOY approach exponential form (68) of $|\psi;t>_{\|}$ cannot be longer considered as the form which reflects accurately the real properties of the system considered.

Eq (66) can be solved for $V_{\|} = V_{\|}(t)$ to obtain

$$\begin{aligned} |\psi;t>_{\|} \ &=\ e^{-itPHP}|\psi>_{\|} \\ &\quad + \sum_{n=1}^{\infty}(-i)^n e^{-itPHP}\int_0^t dt_1 \int_0^{t_1} dt_2 \ldots \\ &\quad \ldots \int_0^{t_{n-1}} dt_n \widetilde{V_{\|}}(t_1)\cdot \ldots \cdot \widetilde{V_{\|}}(t_n)|\psi>_{\|}, \end{aligned} \tag{86}$$

where

$$\widetilde{V_{\|}}(t) = e^{itPHP}V_{\|}(t)e^{-itPHP}. \tag{87}$$

From (86) and (77) using (82) one can conclude that to the lowest non-trivial order

$$V_{\|}^{(1)}(t) = -i\int_0^t PHQe^{-i(t-\tau)QHQ}QHPe^{-i(t-\tau)PHP}\,d\tau. \tag{88}$$

Note that this expression for $V_{\|}^{(1)}(t)$ has been obtained from the exact definition (77) of $V_{\|}$ without using any assumptions considered in [1, 2] and leading to the formula (83) for $V_{\|}^{(1)}$.

From the last formula for $V_{\|}^{(1)}(t)$ and from the expression (83) for $V_{\|}^{(1)}$, which has been obtained with the use of the assumptions exploited in LOY papers, it follows that the conclusion following from the supposition made in [2] that $V_{\|}$ should be time independent can be considered to be justified for $t \gg t_0 = 0$, (that is technically for $t \to \infty$).

3.2 Assumptions leading to the standard form of H_{LOY}.

Analysing the LOY derivation of the effective Hamiltonian discussed one can observe that the components containig the matrix elements $H_{kl}^{(1)}$, $(k, l = 1, 2)$, are neglected in the right sides of the LOY equations equivalent to Eqs(27) in Sec. 2 (see Eqs (18), (19) in [1], or, Eqs (A1.4), (A1.5) in [3], Chap. 5, Appendix 1). The analogous form of Eq (60) can be justified if for every finite $t \geq 0$

$$\| \, PH^{(1)}P|\psi; t >_{\|} \| \; \ll \; \| \, PH^{(1)}Q|\psi; t >_{\perp} \| \, . \tag{89}$$

(This condition replaces the earlier one (25) used in Sec. 2). Assuming that inequality (89) holds, instead of Eq (60), to a sufficiently good approximation, one can consider the following equation

$$i\frac{\partial}{\partial t}|\psi; t >_{\|} \cong m_0 P|\psi; t >_{\|} + PH^{(1)}Q|\psi; t >_{\perp}, \;\; (t \geq 0). \tag{90}$$

Next, according to the ideas leading to Eqs (66), (67), using (65) this equation should be replaced by

$$i\frac{\partial}{\partial t}|\psi; t >_{\|} \cong \left\{m_0 P + V_{\|}\right\}|\psi; t >_{\|}, \;\; (t \geq 0). \tag{91}$$

From this, one can conclude that if condition (89) is fulfilled and if the supposition adopted from [2] that $V_{\|}$ should be time independent holds then the solution of Eq (60) should have exactly the same exponential form (32) as the solution of the LOY equations (18), (19) in [1], (see [1]), formula (21)),

$$|\psi; t >_{\|} \cong e^{-it(m_0 P + V_{\|})}|\psi; t = 0 >_{\|} \equiv e^{-it(m_0 + V_{\|})}|\psi >_{\|} \, . \tag{92}$$

Similarly to the Eq (79) and according to the taken assumptions, such a form of solution of Eq (60) generates the suitable operator $V_{\|}$,

$$V_{\|} = -i \lim_{t \to \infty} \int_0^t PH^{(1)}Qe^{-i(t-\tau)QHQ}QH^{(1)}Pe^{i(t-\tau)(m_0 + V_{\|})} \, d\tau. \tag{93}$$

This expression, by relations (80), (82), to the lowest nontrivial order, gives

$$V_{\|}^{(1)} = -\Sigma(m_0), \tag{94}$$

where

$$\begin{aligned} \Sigma(x) &= PHQ\frac{1}{QHQ - x - i0}QHP \\ &\equiv PH^{(1)}Q\frac{1}{QHQ - x - i0}QH^{(1)}P. \end{aligned} \tag{95}$$

Relations (94) and (84) define the effective Hamiltonian $H_{\|}^{(1)}$, which coincides with H_{LOY}. So, one can write

$$H_{LOY} = m_0 P - \Sigma(m_0). \tag{96}$$

From the course of the derivation of this effective Hamiltonian it follows that such an identification of $H_{\|}^{(1)}$ with H_{LOY} is justifiable. One should stress that such an approximation for H_{LOY} can be considerd as sufficiently good and correct provided that for every $t \geq 0$ the requirement (89) holds.

There is an insignificant difference between this H_{LOY} and $H_{LOY}^{(0)}$ (46) derived in Sec. 2. It occurs because the exact solution (74) of Eqs (61), (62) was used, when the formula was derived for $V_{\|}$ in this Section, contrary to the case of $H_{LOY}^{(0)}$, where the approximate solutions (36) of Eq(19), corresponding to Eq (61) were used.

Note that in the case of H_{LOY} considered, and CPT symmetry conserved, assumptions (54) — (57) imply

$$h_{11}^{LOY} = h_{22}^{LOY}, \tag{97}$$

where $h_{jk}^{LOY} =< \mathbf{j}|H_{LOY}|\mathbf{k} >, \quad j,k = 1,2$,i.e., exactly as for $H_{LOY}^{(0)}$ discussed in Sec. 2 (see (58)).

3.3 Improved H_{LOY}.

Let us consider in detail some implications of the main assumption of the LOY theory, i.e., the relation (92), which is equivalent to (32), (31) in Sec, 2 and (21) in [1]. This relation and similar ones are a direct consequence of the assumption (89) and the other ones of this type.

Note that the assumption on time independence of $V_{\|}$ adopted from [2], the relation (65) and the initial condition (63) states that

$$V_{\|}|\psi;t=0>_{\|} = PHQ|\psi;t=0>_{\perp}\equiv 0, \quad (98)$$
$$V_{\|}|\psi;t>0>_{\|} = PHQ|\psi;t>0>_{\perp}\neq 0. \quad (99)$$

The result (98) means that neglecting the component $PH^{(1)}P|\psi;t>_{\|}$ in the right side of Eq (90) and keeping all the remaining assumptions used in [1, 2] lead, by the relation (92), to the trivial form for the $|\psi;t>_{\|}$:

$$|\psi;t>_{\|}= e^{-it(m_0P+V_{\|})}|\psi;t=0>_{\|}\equiv e^{-itm_0}|\psi>_{\|}, \quad (100)$$

which does not reflect the real processes occuring, e.g., in the neutral kaon complex. In other words, the assumptions of type (89), the only ones under which H_{LOY} can be derived, force two state unstable system considered to behave as one state (one level) stationary subsystem. Thus, the substitutions of type (92) into the Eq (76), or Eq (78), i.e., the Eq (93) can not result in the approximate effective Hamiltonian (84) which could describe correctly the real properties of a two state unstable subsystem. What is more, if one takes into account the analysis performed at the end of Sec. 3.1), formulae (85), (88) then such a conclusion seems to be quite obvious.

A detailed analysis of the assumption (89) permitting the approximate effective Hamiltonian governing the time evolution in two dimensional subspace of states to be of the LOY form (96) indicates that such an assumption cannot be fulfilled for every $t \geq 0$. One finds that at $t = 0$, and thus at $0 < t \to 0$ it is not satisfied. Namely, it is not consistent with the initial condition (63). From (63) it follows that $PHQ|\psi;t = 0>_{\perp}= 0$, and thus $PHQ|\psi;t\to 0>_{\perp}\simeq 0$, which, if (89) holds, leads to the irrational conclusion that at $t = 0$ there should be

$$||PH^{(1)}P|\psi;t=0>_{\|}|| \ll 0.$$

So, keeping in mind that $\| |\psi;t=0>_{\|}\|= 1$ one concludes that there must be $\| PH^{(1)}P|\psi;t\to 0>_{\|}\| > \| PH^{(1)}Q|\psi;t\to 0>_{\perp}\|\simeq 0$ for $0 < t \to 0$ instead of (89). If (74) is inserted into (89) then one can see that the condition (89) cannot be fulfilled for $t \gg t_0 = 0$ either. This means that the derivation of H_{LOY} is incoherent. (The same conclusion refers to all derivated formulae for the LOY effective Hamiltonian in the literature, including [1] –[5], where

the approximations equivalent to the assumption (89) were used). On the one hand, in the LOY treatment of time evolution in a two state subspace initial conditions are defined for $t = t_0 \equiv 0$ and solutions of approximate equations of Eq (90) type are discussed for $t \geq t_0 = 0$, up to $t = +\infty$. On the other hand, within this treatment the approximation of type (89) is used and this approximation in not true for the whole domain of the parameter t, but only for its part (for $t \gg t = t_0 = 0$). In other words, conditions of the (89) type can never reflect the real properties of time evolution in the two state subsystem considered. Therefore H_{LOY} derived within the use of this condition and the assumption of time independece of $V_{\|}$ is unable to describe correctly all the real properties of the system under considerations.

The defects of the LOY method described above can be easily rectified. It is sufficient to abandon this questionable condition (89). In other words, instead of approximate equations of type (90) one should use equations of the type (59), (60) containing component $PH^{(1)}P|\psi, t >_{\|}$, (or, matrix elements H_{jk}, $(j, k = 1, 2)$ in the case of equations of the type (18)). Thus, the exponential form of $|\psi, t >_{\|}$ given by the relation (92) cannot be considered at all, but only the the formula (88) for $V_{\|}(t)$ should be used.

So, let us use the above mentioned improvements of the LOY method and find the approximate $V_{\|}^{(1)} \equiv V_{\|}^{Imp}$ by means of the formula (83) for $PHP = m_0 P + PH^{(1)}P$, which can be derived from (88). In such a case one finds

$$V_{\|}^{(1)} \equiv V_{\|}^{Imp} = -i \lim_{t\to\infty} \int_0^t \Big\{ PH^{(1)} Q e^{-i(t-\tau)(QHQ - m_0)} QH^{(1)}P \times \times e^{i(t-\tau)PH^{(1)}P} \Big\} d\tau. \quad (101)$$

To evaluate this integral it is necessary to calculate $\exp[itPH^{(1)}P]$. Keeping in mind that $PH^{(1)}P$ is the hermitian (2×2) matrix and using the Pauli matrices representation

$$PH^{(1)}P \equiv h_0^{(1)} I_{\|} + \mathbf{h}^{(1)} \cdot \mathbf{s}, \quad (102)$$

where $\mathbf{h}^{(1)}$ and $\mathbf{s}$ denote the following vectors: $\mathbf{h}^{(1)} = (h_x^{(1)}, h_y^{(1)}, h_z^{(1)})$, $\mathbf{s} = (\sigma_x, \sigma_y, \sigma_z)$, and $I_{\|}$ is the unit operator in $\mathcal{H}_{\|}$, and, of course, $I_{\|} \equiv P$,

$$\mathbf{h}^{(1)} \cdot \mathbf{s} = h_x^{(1)} \sigma_x + h_y^{(1)} \sigma_y + h_z^{(1)} \sigma_z, \quad (103)$$

$$h_0^{(1)} = \frac{1}{2}[H_{11}^{(1)} + H_{22}^{(1)}],$$
$$h_z^{(1)} = \frac{1}{2}[H_{11}^{(1)} - H_{22}^{(1)}],$$

$$\begin{aligned}(\kappa^{(1)})^2 \stackrel{\text{def}}{=} \mathbf{h}^{(1)} \cdot \mathbf{h}^{(1)} &= (h_x^{(1)})^2 + (h_y^{(1)})^2 + (h_z^{(1)})^2 \\ &\equiv H_{12}^{(1)} H_{21}^{(1)} + (h_z^{(1)})^2,\end{aligned}$$

(σ_k, $(k = x, y, z)$, are the Pauli matrices), one finds

$$e^{\pm itPH^{(1)}P} = e^{\pm ith_0^{(1)}}\Big[I_{\|}\cos(t\kappa^{(1)}) \pm i\frac{\mathbf{h}^{(1)}\cdot\mathbf{s}}{\kappa^{(1)}}\sin(t\kappa^{(1)})\Big]. \tag{104}$$

It is conveniet to use (102) again and replace $\mathbf{h}^{(1)}\cdot\mathbf{s}$ by $\mathbf{h}^{(1)}\cdot\mathbf{s} = PH^{(1)}P - h_0^{(1)}P$ in Eq (104), which, after some algebra, gives

$$\begin{aligned}e^{+itPH^{(1)}P} &\equiv \frac{1}{2}e^{it(h_0^{(1)}+\kappa^{(1)})}[(1-\frac{h_0^{(1)}}{\kappa^{(1)}})P + \frac{1}{\kappa^{(1)}}PH^{(1)}P] \\ &+ \frac{1}{2}e^{it(h_0^{(1)}-\kappa^{(1)})}[(1+\frac{h_0^{(1)}}{\kappa^{(1)}})P - \frac{1}{\kappa^{(1)}}PH^{(1)}P].\end{aligned} \tag{105}$$

Now, inserting (105) into (101) yields

$$\begin{aligned}V_{\|}^{Imp} &= -\frac{1}{2}\Sigma(m_0 + h_0^{(1)} + \kappa^{(1)})\Big[(1-\frac{h_0^{(1)}}{\kappa^{(1)}})P + \frac{1}{\kappa^{(1)}}PH^{(1)}P\Big] \\ &\quad -\frac{1}{2}\Sigma(m_0 + h_0^{(1)} - \kappa^{(1)})\Big[(1+\frac{h_0^{(1)}}{\kappa^{(1)}})P - \frac{1}{\kappa^{(1)}}PH^{(1)}P\Big].\end{aligned} \tag{106}$$

This means (by (84)) that the improved LOY method leads to the following effective Hamiltonian H_{LOY}^{Imp} governing the time evolution in the two state subspace,

$$H_{LOY}^{Imp} = m_0 P + PH^{(1)}P + V_{\|}^{Imp}. \tag{107}$$

This effective Hamiltonian H_{LOY}^{Imp} differs significantly from the standard expression (46) for $H_{LOY}^{(0)}$ and from (96). The properties of the matrix elements of these effective Hamiltonians, both of which are calculated for the CPT invariant system (57), (56), are the main and the most conspicuous difference. This main difference can be found by comparing standard formula (42) for

matrix elements $h^{LOY(0)}_{jk}$ of $H^{(0)}_{LOY}$ with the formulae for matrix elements h^{Imp}_{jk} of H^{Imp}_{LOY},

$$h^{Imp}_{jk} =< \mathbf{j}|H^{Imp}_{LOY}|\mathbf{k} >= m_0\delta_{jk} + H^{(1)}_{jk} + v^{Imp}_{jk}, \quad (_{j,k=1,2}), \tag{108}$$

where,

$$\begin{aligned} v^{Imp}_{j1} = \; & - \; \frac{1}{2}\Big(1 + \frac{h^{(1)}_z}{\kappa^{(1)}}\Big)\Sigma_{j1}(m_0 + h^{(1)}_0 + \kappa^{(1)}) \\ & - \; \frac{1}{2}\Big(1 - \frac{h^{(11)}_z}{\kappa^{(1)}}\Big)\Sigma_{j1}(m_0 + h^{(1)}_0 - \kappa^{(1)}) \\ & - \; \frac{H^{(1)}_{21}}{2\kappa^{(1)}}\Sigma_{j2}(m_0 + h^{(1)}_0 + \kappa^{(1)}) + \frac{H^{(1)}_{21}}{2\kappa^{(1)}}\Sigma_{j2}(m_0 + h^{(1)}_0 - \kappa^{(1)}), \end{aligned} \tag{109}$$

$$\begin{aligned} v^{Imp}_{j2} = \; & - \; \frac{1}{2}\Big(1 - \frac{h^{(1)}_z}{\kappa^{(1)}}\Big)\Sigma_{j2}(m_0 + h^{(1)}_0 + \kappa^{(1)}) \\ & - \; \frac{1}{2}\Big(1 + \frac{h^{(1)}_z}{\kappa^{(1)}}\Big)\Sigma_{j2}(m_0 + h^{(1)}_0 - \kappa^{(1)}) \\ & - \; \frac{H^{(1)}_{12}}{2\kappa^{(1)}}\Sigma_{j1}(m_0 + h^{(1)}_0 + \kappa^{(1)}) + \frac{H^{(1)}_{12}}{2\kappa^{(1)}}\Sigma_{j1}(m_0 + h^{(1)}_0 - \kappa^{(1)}), \end{aligned} \tag{110}$$

$\Sigma_{jk}(\varepsilon) =< \mathbf{j} \mid \Sigma(\varepsilon) \mid \mathbf{k} >$, and $j, k = 1, 2$. Now, using (52) — (57) it is not difficult to conclude from (108) — (110) that for the CPT invariant but CP noninvariant system, it must be

$$[\Theta, H] = 0 \;\; \Rightarrow \;\; h^{Imp}_{11} \neq h^{Imp}_{22}, \tag{111}$$

contrary to the standard LOY result (58). It should be emphasized in this place that improving the LOY method, only the consistency of the initial conditions (12) and (63) (or (20) and (21)) for the problem with the approximations used (64), (69) and with the geometry (the dimension) of $\mathcal{H}_{\|}$ has been taken into account much more rigorously than it was made by Lee, Oehme and Yang. All steps leading to the formulae for H^{Imp}_{LOY} are well founded and do not impair the main ideas of the standard LOY method. So, the H^{Imp}_{LOY} should reflect the real properties of the system considered much better than it is possible within the use of the standard LOY effective Hamiltonian (42), (46). In this context, the result (111) seems to have serious consequences when interpreting CPT invariance tests, e.g., for the neutral kaon complex [34].

4 Effective Hamiltonian $H_{\|}$ for three state complex.

Using the LOY method the effective Hamiltonian $H_{\|}$ governing the time evolution in n–dimensional subspace $\mathcal{H}_{\|}$ of state space $\mathcal{H}$ for $n > 2$ can also be found. A derivation of such a $H_{\|}$ is rather time consuming when one uses the standard LOY approximation and considers equations of type (18), (19) for amplitudes $a_j(t)$, $(j = 1, 2, \ldots, n)$, (3). On the other hand, such a purpose can be realized relatively easy if one applies the improved LOY method used in Sec. 3 and uses Eqs (59), (61) for components $|\psi; t >_{\|}$, $|\psi; t >_{\perp}$, (49), (51), of a state vector $|\psi; t > \in \mathcal{H}$ instead of the mentioned equations for amplitudes $a_j(t)$. These equations together with the initial condition (63) and assumptions (64), (65) lead to the Equation (79) for $V_{\|}$ and thus, by (80), (82), (similarly to the case considered in Subsection 3.3), to the approximate formula (84) for the improved effective Hamiltonian $H_{\|}^{(1)}$. In the case of a three level subsystem this effective Hamiltonian will be denoted as $H_{\|}^{(1)} \equiv H_{\|}^{(3d)}$, and for the operator $V_{\|}^{(1)}$ defining the $H_{\|}^{(3d)}$ the symbol $V_{\|}^{(3d)}$ will be used. Considering the general case described by Eqs (59), (61), and using (83) one finds

$$V_{\|}^{(1)} \equiv V_{\|}^{(3d)} = -i \lim_{t\to\infty} \int_0^t \Big\{ PHQe^{-i(t-\tau)QHQ} QHP \times \times e^{i(t-\tau)PHP} \Big\} d\tau, \tag{112}$$

and thus (according to (84))

$$H_{\|}^{(3d)} = PHP + V_{\|}^{(3d)}. \tag{113}$$

So, the only problem is to calculate $\exp[itPHP]$ in (112) for the case of $\dim(\mathcal{H}_{\|}) = 3$.

Let the subspace $\mathcal{H}_{\|}$ be spanned by a set of orthonormal vectors $\{|\mathbf{e}_j > \}_{j=1,2,3.} \in \mathcal{H}$, $< \mathbf{e}_j|\mathbf{e}_k >= \delta_{jk}$. Then the projection operator P defining this subspace (see (48)) can be expressed as follows

$$P = \sum_{j=1,2,3} |\mathbf{e}_j >< \mathbf{e}_j| \equiv I_{\|}^{(3d)}, \tag{114}$$

where $I_{\|}^{(3d)}$ is the unity for the three dimensional subspace $\mathcal{H}_{\|}$ considered, and the complementary projector Q, (44), equals $Q = I - P$.

The operator PHP is selfadjoint, so the (3×3) matrix representing PHP in the subspace $\mathcal{H}_{\|}$ is Hermitian matrix. Solving the eigenvalue problem for this matrix,

$$PHP|\lambda_j>=\lambda_j|\lambda_j>,\ \ (_{j=1,2,3}), \tag{115}$$

one obtains the eigenvalues $\lambda_j=\lambda_j^*$, and eigenvectors $|\lambda_j>$, $(j=1,2,3)$. For simplicity we assume that $\lambda_1\neq\lambda_2\neq\lambda_3\neq\lambda_1$, i.e., that all $|\lambda_j>$ are orthogonal,

$$<\lambda_j|\lambda_k>=<\lambda_j|\lambda_j>\delta_{jk},\ \ (_{j,k=1,2,3}). \tag{116}$$

By means of these eigenvectors one can define new projection operators,

$$P_j\stackrel{\rm def}{=}\frac{1}{<\lambda_j|\lambda_j>}|\lambda_j><\lambda_j|,\ \ (_{j=1,2,3}). \tag{117}$$

The property (116) of the solution of the eigenvalue problem for PHP considered implies that

$$P_jP_k=P_j\delta_{jk},\ \ (_{j=1,2,3}), \tag{118}$$

and that the completeness requirement for the subspace $\mathcal{H}_{\|}$

$$\sum_{j=1,2,3}P_j=P, \tag{119}$$

holds. Now, using the projectors P_j one can write

$$PHP=\sum_{j=1,2,3}\lambda_jP_j, \tag{120}$$

and

$$Pe^{+itPHP}=P\sum_{j=1,2,3}e^{+it\lambda_j}P_j. \tag{121}$$

This last relation is the solution for the problem of finding $\exp[itPHP]$ and leads to the following formula for $V_{\|}^{(3d)}$,

$$V_{\|}^{(3d)}=-i\lim_{t\to\infty}\sum_{j=1,2,3}\int_0^t PHQe^{-i(t-\tau)(QHQ-\lambda_j)}QHP\,d\tau\,P_j. \tag{122}$$

A computation of the value of this integral can be easy performed and yields

$$V_{\|}^{(3d)}=-\sum_{j=1,2,3}\Sigma(\lambda_j)P_j, \tag{123}$$

(where $\Sigma(\lambda)$ is defined by the formula (95)), which by (113) solves the problem of finding the improved LOY effective Hamiltonian governing the time evolution in the three state subspace $\mathcal{H}_{\|}$ of the total state space $\mathcal{H}$.

The results obtained in this Section can be easily generalized to the case of $\dim(\mathcal{H}_{\|}) = n > 3$.

5 Final remarks.

Detailed analysis of assumptions leading to the standard form of the LOY effective Hamiltonian governing the time evolution in a two state subsystem indicates that some assumptions, which have been used in the LOY treatment of the problem, and which the WW theory of single line width uses, should not be directly applied to the case of two, or more, level subsystems interacting with the rest of the physical system considered. Namely, when one considers the single line width problem in the WW manner it is quite sufficient to analyse the smallness of matrix elements of the interaction Hamiltonian, $H^{(1)}$, only. For the multilevel problem, contrary to the single line problem, such a smallness does not ensure the suitable smallness of components of the evolution equations containing these matrix elements. Moreover, there is no necessity of taking into account the internal dynamics of the subsystem, which also has an effect on the widths of levels in many levels subsystems, in such a case. The observed level widths in two and more level subsystem depend on the interactions of this subsystem with the rest, but they also depend on the interactions between the levels forming this subsystem. So, the internal interactions in the subsystem considered cannot be neglected when one wants to describe the real properties of multi state subsystems.

From the form of Eqs (18) — (20) in [1] (or, Eqs (A1.) — (A1.6) in [3], Appendix 1 of Chap. 5) it follows that the LOY and related treatments of time evolution in two state subsystem use the WW theory of the single line width without any modification of the questionable points of the WW method and do not consider at all the aspects of time evolution in many state subsystem mentioned above. When one wants to apply the LOY method of searching for the properties of the time evolution in a two level subsystem, in order to be more rigorous than it was done in [1] — [9] and than it is possible within the standard WW approach, one should replace requirements (24) —

(26) by the following ones

$$|\sum_{J,\varepsilon} F_J(\varepsilon;t)H^{(1)}_{kJ}(\varepsilon)| \;\ll\; m_0|a_k(t)|, \quad (k{=}1{,}2), \tag{124}$$

$$|\sum_{l=1,2} H^{(1)}_{kl}a_l(t)| \;\ll\; |\sum_{J,\varepsilon} F_J(\varepsilon;t)H^{(1)}_{kJ}(\varepsilon)|, \quad (k{=}1{,}2), \tag{125}$$

and

$$|\sum_{L,\varepsilon'} F_L(\varepsilon';t)H^{(1)}_{J,L}(\varepsilon,\varepsilon')| \ll |\sum_{k=1,2} H^{(1)}_{Jk}(\varepsilon)a_k(t)|. \tag{126}$$

Such a form of assumptions replacing (24) — (26) enable, e.g., to detect the inconsistencies between the main LOY assumption (31) (or, (32)) and the initial condition (21). From (21) it follows that the requirments of type (125), the only ones under which the approximate Eqs (27) are sufficiently accurate, can not be fulfilled for $t = 0$ and for $t \to 0$. (It is impossible to draw a similar conclusions from the assumptions of type (25)). So, the expected and assumed exponential form, (32), of $|\psi;t>_{\|}$, (33), should take into account the fact that for short t the influence of $H^{(1)}$ on the form of $|\psi;t>_{\|}$ predominates over the the influence coming from the component containing $\sum_{J,\varepsilon} F_J(\varepsilon;t)$ in Eq (18). The influence of this last component can become crucial only for suficiently large times $t \geq T_{as} > 0$. It seems to be obvious that $|\psi;T_{as}>_{\|} \neq |\psi;t=t_0=0>_{\|}$. So, whether one should replace $|\psi;t=0>_{\|}$ by $|\psi;t=T_{as}>_{\|}$ in the assumption (32), or one should leave $|\psi;t=0>_{\|}$ unchanged in (32) but change the index of the power in (32) adding $H^{(1)}$, cut down to the subspace $\mathcal{H}_{\|}$, there. These cases, similarly to the improved LOY method used in Subsection 3.3, lead to the effective Hamiltonian $H_{\|} = H^{Imp}_{LOY}$, which differes from the standard LOY effective Hamiltonian $H^{(0)}_{LOY}$, (42), (46).

Analysing the standard derivation of H_{LOY} [1, 3] one can draw a conclusion which seems to be strange that conditions of type (24) — (26), necessary to obtain this H_{LOY}, lead to the same form of the efective Hamiltonian, $H_{\|}$, governing the time evolution in subspace $\mathcal{H}_{\|}$ independently of the dimension of this $\mathcal{H}_{\|}$. This means that the properties of the subsystem considered which manifests themselves during the time evolution, should not depend on the dimension of the subspace of states of this subsystem. The common form of $H_{\|}$ is given by (46) and (96) and this is the form which can be obtained by means of the improved LOY method only for one–dimensional subspace $\mathcal{H}_{\|}$.

Taking

$$P \equiv |\psi><\psi| \stackrel{\rm def}{=} P_\psi, \;\; Q = I - P_\psi, \tag{127}$$

where $<\psi|\psi>= 1$, one has

$$PHP =<\psi|H|\psi> P_\psi, \tag{128}$$

and thus, using (83) one can calculate $V_\parallel^{(1)} \stackrel{\rm def}{=} V_\parallel^{(1d)}$, which equals

$$V_\parallel^{(1d)} = -\Sigma_\psi(<\psi|H|\psi>), \tag{129}$$

where $\Sigma_\psi(x)$ is defined by the relation (95) for $P \equiv P_\psi$. So, the approximate effective Hamiltonian, $H_\parallel^{(1)} \stackrel{\rm def}{=} H_\parallel^{(1d)}$, for the case $\dim(\mathcal{H}_\parallel) = 1$ appears to be (see (84)),

$$H_\parallel^{(1d)} = P_\psi H P_\psi + V_\parallel^{(1d)}. \tag{130}$$

Such a form of $V_\parallel^{(1)}$, and thus of the effective Hamiltonian $H_\parallel^{(1)}$, is produced by the standard LOY approach for the case of the arbitrary dimension of $\mathcal{H}_\parallel$. From the last formula and from the relations (123), (113), and (107) it follows that the form of the effective Hamiltonians obtained within the use of the improved LOY method desribed in Sec. 3 depends on the geometry of the problem, i.e., on the dimension of the subspace $\mathcal{H}_\parallel$. Such an implication of the improved LOY method, (contrary to the result, which can be obtained by the standard LOY method), seems to be quite natural and obvious for the real physical systems. Therefore the improved LOY method, which is consistent with the initial condition for the problem, (21) or (63), and more rigorous than the standard one, should reflect the real properties of the system considered more accurately than it is possible within the use of the LOY theory in its original form.

It seems that the differences between H_{LOY} and H_{LOY}^{Imp} can be explained by means of the resolvent formalism. From the point of view of this formalism H_{LOY} is generated by the pole approximation (see [4, 10, 32] and the paper by Horwitz and Marchand cited in [26]). When the nonorthogonality of the residues in neglected then within this approximation one obtains the effective Hamiltonian H_{LOY}, that is, in general, the effective Hamiltonian of the form (129), (130). In fact, as it has been pointed out in [32] these residues are not orthogonal. Taking into account this fact results in the effective Hamiltonian for the two–channel problem which differs from the standard form of H_{LOY}.

The connection between the approach based on the more exact resolvent formalism and the method described at the beginning of Sec. 3.1) and at the end of that Subsection can be found by the use of the Laplace transforms. Eq. (75) and the relation (77) are the exact ones. There is a one–to–one relation between Eq. (75) for $|\psi; t >_{\|}$ and Equation for $|\widetilde{\psi}; z >_{\|} = \mathcal{L}[|\psi; t >_{\|}](z)$ (here $\mathcal{L}[.](z)$ denotes the Laplace transform) in terms of the reduced resolvent. Solution (86) of the Eq. (66) for $|\psi; t >_{\|}$ can also be considered as the exact one. Formula (88) for $V_{\|}^{(1)}(t)$, from which the expression (83) for the $V_{\|} \simeq V_{\|}^{(1)}$ follows, has been obtained using these exact relations. On the other hand, $V_{\|}^{(1)}$ has been used Sec. 3.3) to obtain H_{LOY}^{Imp}. Therefore the supposition made above that the nonorthogonality of residues is responsible for the difference between the forms of H_{LOY} and H_{LOY}^{Imp} seems to be justified.

Note that, as it has been shown in Sec. 4, the discussed improved method allows one to relatively easy compute the effective Hamiltonian $H_{\|}$ for n–dimensional ($n \geq 2$) subspace $\mathcal{H}$ of states.

The size of the effect (111) taking place for H_{LOY}^{Imp} can be easily estimated for the generalized Fridrichs–Lee model (see Appendix). Within this model one can obtain that

$$(h_{11}^{Imp} - h_{22}^{Imp}) \simeq 0,94 \times 10^{-14} \mathrm{Im}\,(H_{12}), \tag{131}$$

where $H_{jk} =< \mathbf{j}|H|\mathbf{k} >,\ (j,k = 1,2)$. Comparing this estimation and, e.g., the limit $|m_{K_0} - m_{\overline{K}_0}| < 2,0 \times 10^{-18} |m_{K_0}|$ (see formula (121) in [33]) one can conclude that the H_{LOY}^{Imp} does not lead to effects which are in conflict with the results of recent experiments.

From (131) it follows that the effect (111) is very small indeed, and it is beyond the accuracy of today's experiments with neutral kaons. Test of higher accuracy are expected to be performed in the near future [7]. So, there is a chance that these tests will confirm this effect. Nevertheless, the improved formulae for the LOY effective Hamiltonian seem to have a great meaning for the interpretation of some recent theoretical speculations such as those considered, for instance, in [34, 35, 36]. Indeed, the parameters used in [34] to describe the deviations of quantum mechanics, or violations of CPT, are of similar order to (131). This means that the interpretation of CPT tests, or tests of modified quantum mechanics, based on the theory developed in [34] may be incorretct. A similar conclusion seems to be right with reference to theories describing effects of external fields on the neutral

kaon system [35]. Also, the interpretation of tests of special relativity and of the equivalence principle [36] is based on the standard form, (46), of the $H^{(0)}_{LOY}$. The order of the effects discussed in [36] can be compared to (131). So it seems to be obvious that the application of H^{Imp}_{LOY}, (107) instead of $H^{(0)}_{LOY}$, when one considers theories developed in all these papers, can lead to conclusions which need not agree with those obtained in [34, 35, 36].

The last observation is that comparing the formulae for the matrix elements, (108), of the improved LOY effective Hamiltonian, H^{Imp}_{LOY}, with the formulae for the matrix elements of the effective Hamiltonian $H_{\|} \stackrel{\rm def}{=} H_{\|}(t \to \infty)$ derived from the Krolikowski–Rzewuski equation [27] — [29] in [30] and discussed also in [25], one finds that all they are identical. Also, the general formula, (79), (80), for the operator $V^{(1)}_{\|}$ is simply the asymptotic case of the formula for $V^{(1)}_{\|}(t)$ obtained in [30, 31], namely $V^{(1)}_{\|} \equiv \lim_{t\to\infty} V^{(1)}_{\|}(t)$. So, the formalism applied in [25, 30] and also in [17] to describe the properties of the neutral kaon and similar complexes, should not be considered as an alternative approach to the description of time evolution in such complexes. Simply, the formalism mentioned is more rigorous than the improved LOY method, but both these approaches produce the same formulae for the approximate effective Hamiltonians for the problem.

A Appendix

In the generalized Fridrichs–Lee model, the Hamiltonian is given by (see (2.1) in [13], or (3.19) in [14]).

$$\begin{aligned} H = \sum_{j,k=1}^{2} m_{jk} V_j^+ V_k \;\; &+ \;\; \sum_{n=1}^{N} \mu_n N_n^+ N_n + \int_0^\infty d\omega\, \Theta^+(\omega)\Theta(\omega) \\ &+ \int_0^\infty d\omega \sum_{j,n} g_{jn}(\omega) V_j N_n^+ \Theta^+(\omega) \\ &+ \int_0^\infty d\omega \sum_{j,n} g_{nj}(\omega) V_j^+ N_n \Theta(\omega), \end{aligned} \tag{A.1}$$

where $g_{nj}(\omega) = g^*_{jn}(\omega)$. The bare particles are V_1, V_2, N_n $(1 \leq n \leq N)$, and Θ particles. The following "charges" are conserved in this model:

$$\begin{aligned} Q_1 &= \sum_{j=1}^{2} V_j^+ V_j + \sum_{n=1}^{N} N_n^+ N_n, \\ Q_2 &= \sum_{n=1}^{N} N_n^+ N_n - \int d\omega \Theta^+(\omega)\Theta(\omega). \end{aligned}$$

The corresponding eigenvalues will be denoted by q_1 and q_2. The Hilbert space in this model is devided into orthogonal sectors $\mathcal{H}(q_1, q_2)$, each with different assignment of q_1 and q_2 values. Considering the lowest nontrivial sector, where $q_1 = 1$ and $q_2 = 0$ and the bare states are labeled by $|V_j >= V_j^+|0>$, $(j = 1, 2)$, $|n, \omega >= N_n^+\Theta^+(\omega)|0>$, $(n = 1, 2, \ldots, N)$, and then identifying V_1 as K_0 and V_2 as $\overline{K}_0$, after some algebra, one finds [30]

$$\begin{aligned} h_{11}^{Imp} - h_{22}^{Imp} &= \frac{i}{4}\frac{m_{21}\Gamma_{12} - m_{12}\Gamma_{21}}{|m_{12}|} \times \\ &\times\Big\{\frac{(m_0-\mu)^{1/2}}{(m_0-\mu-|m_{12}|)^{1/2}} - \frac{(m_0-\mu)^{1/2}}{(m_0-\mu+|m_{12}|)^{1/2}}\Big\}, \end{aligned} \tag{A.2}$$

where, $\mu \equiv \mu_n$, $(n = 1, 2, \ldots, N)$, and, in the CPT invariant case: $m_0 = H_{11} = H_{22}$, and Γ_{jk}, $(j, k = 1, 2)$ can be defined as follows

$$\Gamma_{jk} f(\lambda) = \pi \sum_{n=1}^{N} g^*_{nj}(\lambda) g_{nk}(\lambda), \tag{A.3}$$

where, for simplicity, the weight function $f(\lambda)$ can be choosen analogously to (3.8) in [13].

Now, following [13, 14] one can identify Γ_{jk}, $(j, k = 1, 2)$ with those appearing in the LOY theory (42), m_0 can be considered as kaon mass [13], $m_{jk} \equiv H_{jk}\,(j, k = 1, 2)$, μ can be treated as the mass of the decay products of the neutral kaon [13]. The additional assumption $|m_{12}| \ll (m_0 - \mu)$ leads to the following estimation for $(h_{11}^{Imp} - h_{22}^{Imp})$:

$$h_{11}^{Imp} - h_{22}^{Imp} \simeq i\frac{m_{21}\Gamma_{12} - m_{12}\Gamma_{21}}{4(m_0 - \mu)} \tag{A.4}$$

An equivalent form of this estimation is the following one:

$$h_{11}^{Imp} - h_{22}^{Imp} = \frac{-\mathrm{Re}(m_{12})\ \mathrm{Im}(\Gamma_{12}) + \mathrm{Im}(m_{12})\ \mathrm{Re}(\Gamma_{12})}{2(m_0 - \mu)}. \qquad (A.5)$$

Real properties of neutral K–complex enable us to conclude that the contribution of $\mathrm{Im}\,(\Gamma_{12})$ in the numerator of (A.5) is neglegibly small in comparison with the contribution of $\mathrm{Re}\,(\Gamma_{12})$ in the considered case of neutral K–mesons [7]. Finally, taking into account that $2\mathrm{Re}\,(\Gamma_{12}) \simeq (\gamma_s - \gamma_l)$ [7], the estimation (A.5) takes the following form:

$$\begin{aligned} h_{11}^{Imp} - h_{22}^{Imp} &= \mathrm{Im}(m_{12})\ \frac{\gamma_s - \gamma_l}{4(m_0 - \mu)} \\ &\approx \mathrm{Im}(m_{12})\ \frac{\gamma_s}{4(m_0 - \mu)}. \end{aligned} \qquad (A.6)$$

For the neutral K–system, to evaluate $(h_{11}^{Imp} - h_{22}^{Imp})$ one can take $\tau_s \simeq 0,89 \times 10^{-10}$sec [37]. Hence $\gamma_s = \frac{\hbar}{\tau_s} \sim 7,4 \times 10^{-12}$MeV and (following [13]) $(m_0 - \mu) = m_K - 2m_\pi \sim 200$ MeV. Thus

$$(h_{11}^{Imp} - h_{22}^{Imp}) \sim 0,93 \times 10^{-14}\mathrm{Im}\,(m_{12}) \equiv 0,93 \times 10^{-14}\mathrm{Im}\,(H_{12}). \qquad (A.7)$$

References

[1] T. D. Lee, R. Oehme and C. N. Yang, Phys. Rev., **106**, (1957) 340.

[2] T. D. Lee and C. S. Wu, Annual Review of Nuclear Science, **16**, (1966) 471.

[3] Ed.: M. K. Gaillard and M. Nikolic, Weak Interactions, (INPN et de Physique des Particules, Paris, 1977); Chapt. 5, Appendix A.

[4] S. M. Bilenkij, Particles and nucleus, vol. 1, No 1 (Dubna 1970), p. 227 [in Russian]. P. K. Kabir, The CP-puzzle, Academic Press, New York 1968.

[5] J. W. Cronin, Rev. Mod. Phys. **53**, (1981) 373. J. W. Cronin, Acta Phys. Polon., **B15**, (1984) 419. V. V. Barmin, et al., Nucl. Phys. **B247**, (1984) 293. L. Lavoura, Ann. Phys. (NY), **207**, (1991) 428. C. Buchanan, et al., Phys. Rev. **D45**, (1992) 4088. C. O. Dib, and R. D. Peccei, Phys. Rev.,**D46**, (1992) 2265. R. D. Peccei, CP and CPT Violation: Status and Prospects, Preprint UCLA/93/TEP/19, University of California, June 1993.

[6] E. D. Comins and P. H. Bucksbaum, Weak interactions of Leptons and Quarks, (Cambridge University Press, 1983). T. P. Cheng and L. F. Li, Gauge Theory of Elementary Particle Physics, (Clarendon, Oxford 1984).

[7] L. Maiani, in "The Second DaΦne Physics Handbook", vol. 1, Eds. L. Maiani, G. Pancheri and N. Paver, SIS — Pubblicazioni, INFN — LNF, Frascati, 1995; pp. 3 — 26.

[8] L. A. Khalfin, Theory of Unstable Neutrino Mixing and the 17 keV Neutrino Problem, preprint HU–TFT-92–21, Helsinki, June 1992.

[9] M. Baldo–Ceolin, Neutron–antineutron oscillation experiments, Proceedings of the "International Conference of Unified Theories and Their Experimental Tests" — Venice — 16–18 March 1982; Sensitive Search for Neutron–Antineutron Transitions at the Ill Reactor, AIP Conference Proceedings No 125 of the Fifth International Symposium on "Capture Gamma–Ray Spectroscopy and Related Topics" — Edited by S. Raman, Knoxville, September 1984. p.871.

[10] L. A. Khalfin, The theory of K_0,$\overline{K}_0$ (D_0,$\overline{D}_0$ and T_0,$\overline{T}_0$) mesons beyond the Weisskopf-Wigner approximation and the CP–problem, preprint LOMI P–4–80, Leningrad, February 1980.

[11] L. A. Khalfin, Foundations of Physics, **27** (1997) 1549.

[12] L. A. Khalfin, Preprints of the CPT, The University of Texas at Austin: DOE-ER-40200-211, February 1990 and DOE-ER-40200-247, February 1991; (unpublished, cited in [13]), and references one can find therein.

[13] C. B. Chiu and E. C. G. Sudarshan, Phys. Rev. **D 42** (1990) 3712.

[14] E. C. G. Sudarshan, C. B. Chiu and G. Bhamathi, Unstable Systems in Generalized Quantum Theory, preprint DOE-40757-023 and CPP-93-23, University of Texas, October 1993.

[15] K. Urbanowski, Phys. Lett. **B 313**, (1993) 374.

[16] K. Urbanowski, Is the new interpretation of some standard CPT–violation parameters necessary?, Preprint of the Pedagogical University, No WSP–IF 94–39, Zielona Gora, May 1994; CPT transformation properties of the exact effective Hamiltonian for neutral kaon complex, Preprint of the Pedagogical University, No WSP–IF 96–44, Zielona Gora, March 1996; CPT transformation properties of the exact effective Hamiltonian for neutral kaon and similar complexes, Preprint of the Pedagogical University, No WSP–IF 97–50, Zielona Gora, July 1997 – **hep–ph/9803376**.

[17] K. Urbanowski, Int. J. Mod. Phys. **A 13**, (1998), 965.
[18] K. Urbanowski, Why the LOY model cannot be used for designing CPT–invariance tests in neutral kaons systems? On CPT–noninvariance of systems with exponentially decaying particles, preprint of the Pedagogical University No WSP–IF 94–38, Zielona Gora, May 1994.
[19] T. Mochizuki, N. Hashimoto, A. Shinobori, S. Y. Tsai, Remarks on Theoretical Frameworks Describing The Neutral Kaon System, preprint Of The Nihon University, No NUP–A–97–14, Tokyo, June 1997.
[20] V. F. Weisskopf and E. T. Wigner, Z. Phys. **63** (1930) 54; **65** (1930) 18.
[21] P. K. Kabir and A. Pilaftsis, Phys. Rev. **A 53**, (1966) 66.
[22] W. Pauli, in: "Niels Bohr and the Developmnet of Physics". ed. W. Pauli (pergamon Press, London, 1955), pp. 30 — 51. G. Luders, Ann. Phys. (NY) **2** (1957) 1. . R. Jost, Helv. Phys. Acta **30** (1957) 409.R.F. Streater and A. S. Wightman, "CPT, Spin, Statistics and All That" (Benjamin, New York, 1964). N. N. Bogolubov, A. A. Logunov and I. T. Todorov, "Introduction to Axiomatic Field Theory" (Benjamin, New York, 1975).
[23] A. Messiah, Quantum Mechanics, vol. 2, (Wiley, New York 1966).
[24] A. Bohm, Quantum Mechanics: Foundations and Applications, 2nd ed., (Springer, New York 1986).
[25] K. Urbanowski, Int. J. Mod. Phys. **A 10**, (1995) 1151.
[26] e.g., R. Zwanzig, Physica **30**, (1964), 1109. F. Haake, "Statistical Treatment of Open Systems by Generalized Master Equations", Springer Tracts in Modern Physics, Vol. 66, (Springer, Berlin, 1973). L. P. Horwitz and J. P. Marchand, Rocky Mount. J. Math. **1**, (1971), 225. E. B. Davis, "Quantum Theory of Open System", (Academic, London, 1976).
[27] W. Krolikowski and J. Rzewuski, Bull. Acad. Polon. Sci. **4** (1956) 19.
[28] W. Krolikowski and J. Rzewuski, Nuovo. Cim. **B 25** (1975) 739 and refernces therein.
[29] K. Urbanowski, Acta Phys. Polon. **B 14** (1983) 485.
[30] K. Urbanowski, Int. J. Mod. Phys. **A 8**, (1993)3721.
[31] K. Urbanowski, Phys. Rev. **A 50**, (1994) 2847.
[32] E. Cohen and L. P. Horwitz, Preprints No **hep–ph/9808030, hep–ph/9811332**.
[33] L. K. Gibbons et al., CP and CPT Symmetry Tests from the Two–pion Decays of the Neutral Kaon with the Fermilab–E731 Detector, preprints FERMILAB–Pub–95/392–E, EFI–95–76; Phys.Rev. **D55**, (1997), 6625.

[34] J. Ellis, J.L. Lopez, N.E. Mavratos and D.V. Nanopulous, Phys. Rev. **D53**, (1996), 3846; J. Ellis, N.E. Mavratos and D.V. Nanopulous, Phys. Lett., **B 293**,(1992), 142; V.A. Kostelecky, Phys. rev. Lett., **80**, (1998), 1818.

[35] E. Fischbach, C. Talmadge, Nature, **356**, (1992), 207; E. Fischbach, D. Sudarsky, A. Szafer, C. Talmadge and S. H. Aronson,Phys. Rev. Lett., **56**, (1986), 3; D. Sudarsky, E. Fischbach, C. Talmadge, A. H. Aronson and H.–Y. Cheng, Annals of Physics, **207**, (1991), 103; E. Fischbach, C. Talmadge, Preprint No **hep–ph/9606249**.

[36] T. Haymbye, R. B. Mann, U. Sarkar, Phys. Lett., **B421**, (1998), 105; T. Haymbye, R. B. Mann, U. Sarkar, Phys. Rev. **D58**, (1998), art. no 025003; C. Alvarez, R. B. Mann, Phys. Rev., **D55**, (1997), 1732; R. J. Hughes, Phys. Rev., **46**, (1992), R2283; I. R. Kenyon, Phys. Lett., **B 237**, (1990), 274.

[37] Review of Particle Physics, Phys. Rev. **D54**, (1996), No 1, Part 1.